Electrical Circuits (2)

Lecture 4

Parallel Resonance and its Filters

Dr.Eng. Basem ElHalawany

Parallel Resonance Circuit

It is usually called tank circuit

Ideal Circuits

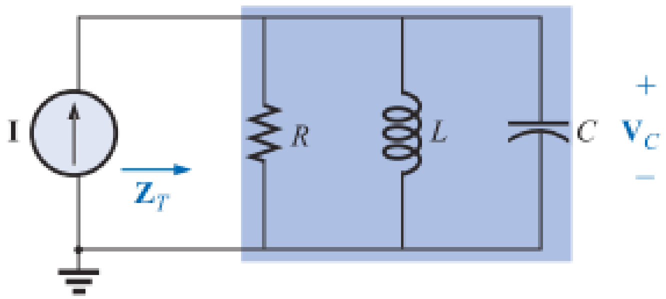


FIG. 20.21
Ideal parallel resonant network.

Practical Circuits

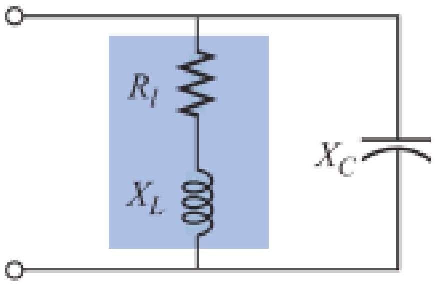
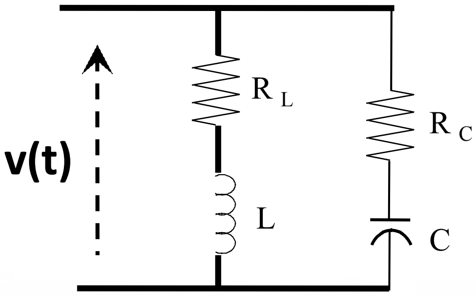


FIG. 20.22
Practical parallel L-C network.

Complex Configuration



Ideal Parallel Resonance Circuit

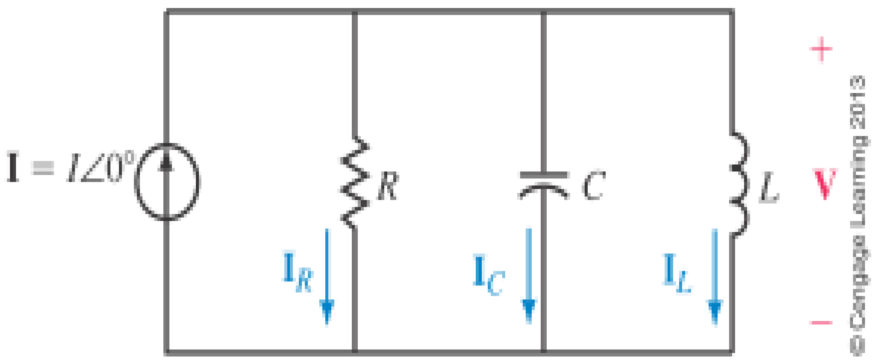
The total admittance

$$Y = Y_1 + Y_2 + Y_3$$

$$Y = \frac{1}{R} + \frac{1}{(j\omega L)} + \frac{1}{(-j/\omega C)}$$

$$Y = \frac{1}{R} + \frac{-j}{\omega L} + j\omega C$$

$$Y = \frac{1}{R} + j(\omega C - 1/\omega L)$$



Condition for Ideal Parallel Resonance

Resonance occurs when the imaginary part of **Y** is zero

$$\omega C - \frac{1}{\omega L} = 0$$

$$X_C = X_L.$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/s}$$

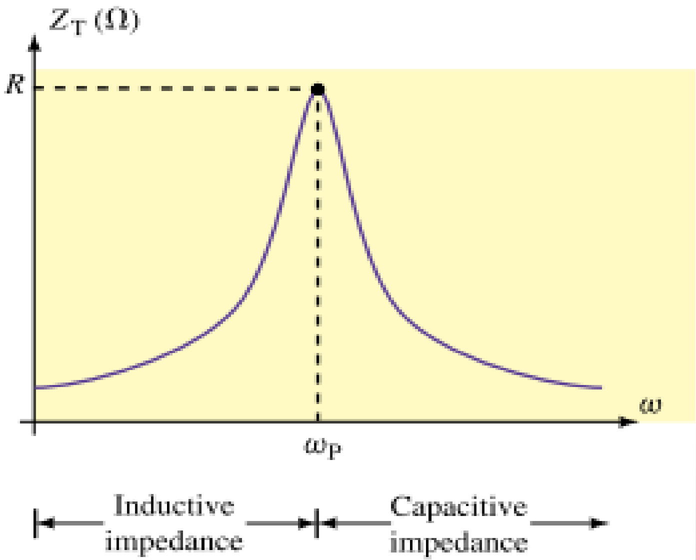
Ideal Parallel Resonance Circuit

At parallel resonance:

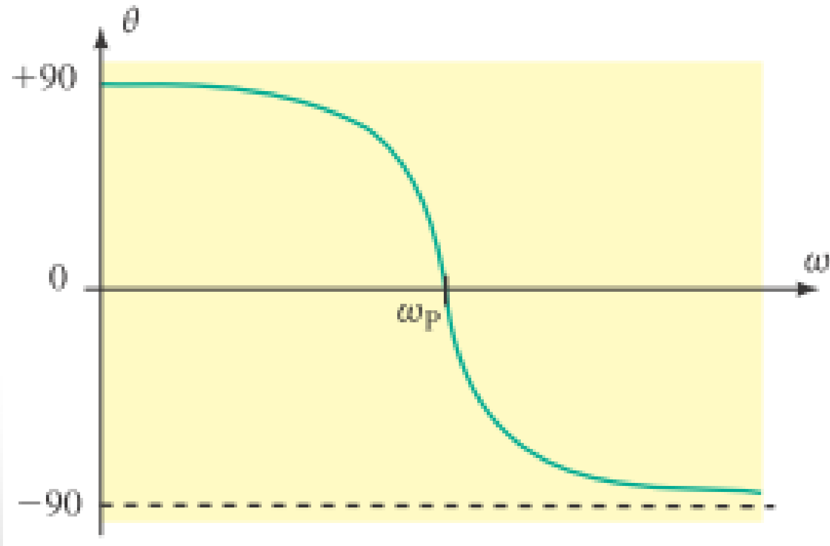
- ✓ At resonance, the admittance consists only conductance G = 1/R.
- ✓ The value of current will be minimum since the total admittance is minimum.
- ✓ The voltage and current are in phase (Power factor is unity).
- ✓ The inductor and capacitor reactances cancel, resulting in a circuit voltage simply determined by Ohm's law as:

$$\mathbf{V} = \mathbf{I}R = IR\angle 0^\circ$$

- ✓ The frequency response of the impedance of the parallel circuit is shown



exactly opposite to that in series resonant circuits,



Ideal Parallel Resonance Circuit

The Q of the parallel circuit is determined from the definition as

$$Q_P = \frac{\text{reactive power}}{\text{average power}} = \frac{V^2/X_L}{V^2/R}$$

$$Q_P = \frac{R}{X_{LP}} = \frac{R}{X_C}$$

Reciprocal of series

The current

$$\mathbf{I}_R = \frac{\mathbf{V}}{\mathbf{R}} = \mathbf{I}$$

$$\mathbf{I}_L = \frac{\mathbf{V}}{X_L\angle 90^\circ} = \frac{V}{R/Q_P}\angle -90^\circ = Q_P I\angle -90^\circ$$

$$\mathbf{I}_C = \frac{\mathbf{V}}{X_C\angle -90^\circ} = \frac{V}{R/Q_P}\angle 90^\circ = Q_P I\angle 90^\circ$$

- ✓ The currents through the inductor and the capacitor have the same magnitudes but are 180 out of phase.
- ✓ Notice that the magnitude of current in the reactive elements at resonance is Q times greater than the applied source current.

Ideal Parallel Resonance Circuit

- Parallel resonant circuit has same parameters as the series resonant circuit.

Resonance frequency:

$$\omega_p = \frac{1}{\sqrt{LC}} \text{ rad/s}$$

Half-power frequencies:

$$\omega_1 = -\frac{1}{2RC} + \sqrt{\frac{1}{4R^2C^2} + \frac{1}{LC}} \quad \text{(rad/s)}$$

$$\omega_2 = \frac{1}{2RC} + \sqrt{\frac{1}{4R^2C^2} + \frac{1}{LC}} \quad \text{(rad/s)}$$

Bandwidth and Q-factor:

$$\text{BW} = \omega_2 - \omega_1 = \frac{1}{RC} \quad \text{(rad/s)}$$

$$\text{BW} = \frac{\omega_P}{R(\omega_P C)} = \frac{X_C}{R}\omega_P$$

$$\text{BW} = \frac{\omega_P}{Q_P} \quad \text{(rad/s)}$$

$$Q = \frac{\omega_0}{B} = \omega_0 RC = \frac{R}{\omega_0 L}$$

$$\omega_1 = \omega_0\sqrt{1 + \left(\frac{1}{2Q}\right)^2} - \frac{\omega_0}{2Q},$$

$$\omega_2 = \omega_0\sqrt{1 + \left(\frac{1}{2Q}\right)^2} + \frac{\omega_0}{2Q}$$

Practical Parallel Resonance Circuit

Effect of Winding Resistance on the Parallel Resonant Frequency

- The internal resistance of the coil must be taken into consideration because it is no longer be included in a simple series or parallel combination with the source resistance and any other resistance added for design purposes.
- Even though RL is usually relatively small in magnitude compared with other resistance and reactance levels of the network, it does have an important impact on the parallel resonant condition,

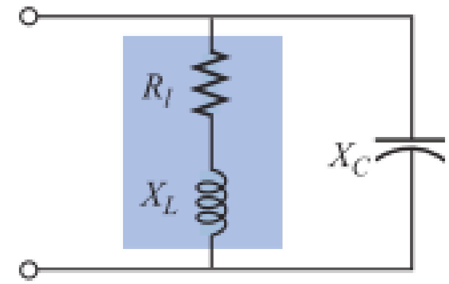


FIG. 20.22
Practical parallel L-C network.

1. Find a parallel network equivalent to the series R-L branch

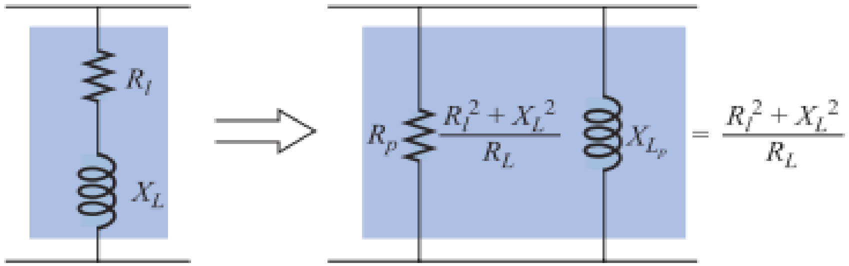


FIG. 20.23
Equivalent parallel network for a series R-L combination.

$$\mathbf{Z}_{R\text{-}L} = R_l + j X_L$$

$$\mathbf{Y}_{R\text{-}L} = \frac{1}{\mathbf{Z}_{R\text{-}L}} = \frac{1}{R_l + j X_L} = \frac{R_l}{R_l^2 + X_L^2} - j \frac{X_L}{R_l^2 + X_L^2}$$

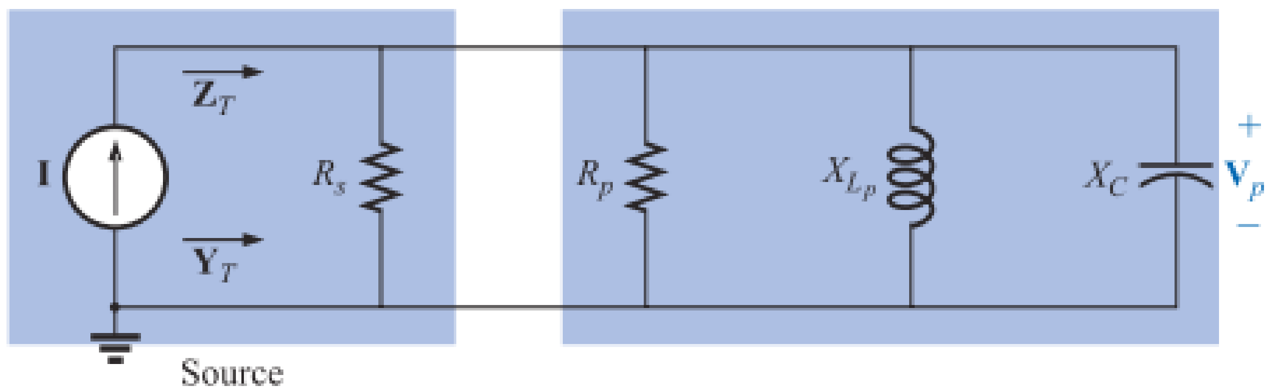
Practical Parallel Resonance Circuit

$$R_p = \frac{R_l^2 + X_L^2}{R_l}$$

$$X_{L_p} = \frac{R_l^2 + X_L^2}{X_L}$$

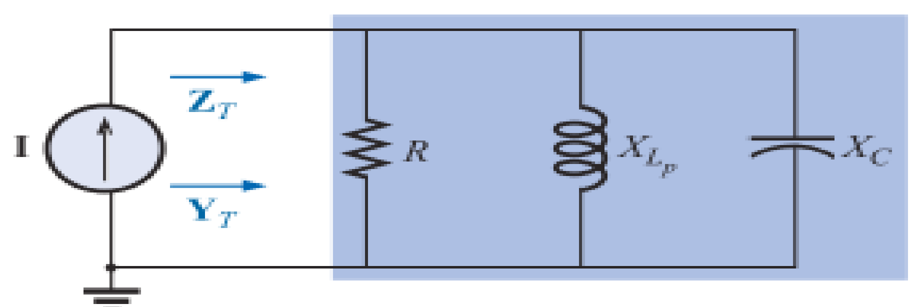
$$\mathbf{Y}_{R\text{-}L} = \frac{1}{\dfrac{R_l^2 + X_L^2}{R_l}} + \frac{1}{j\left(\dfrac{R_l^2 + X_L^2}{X_L}\right)} = \frac{1}{R_p} + \frac{1}{j\,X_{Lp}}$$

Redrawing the network



If we define the parallel combination of R_s and R_p by the notation

$$R = R_s \parallel R_p$$



Practical Parallel Resonance Circuit

$$\mathbf{Y}_T = \frac{1}{R} + j\left(\frac{1}{X_C} - \frac{1}{X_{L_p}}\right)$$

$$\frac{1}{X_C} - \frac{1}{X_{L_p}} = 0$$

$$\frac{1}{X_C} = \frac{1}{X_{L_p}}$$

$$X_{L_p} = X_C$$

$$\frac{R_l^2 + X_L^2}{X_L} = X_C$$

The resonant frequency, fp , can now be determined as follows:

$$R_l^2 + X_L^2 = X_C X_L = \left(\frac{1}{\omega C}\right)\omega L = \frac{L}{C}$$

$$X_L^2 = \frac{L}{C} - R_l^2 \qquad 2\pi f_p L = \sqrt{\frac{L}{C} - R_l^2}$$

$$f_p = \frac{1}{2\pi L}\sqrt{\frac{L}{C} - R_l^2}$$

Multiplying within the square-root sign by C/L and rearranging produces :

$$f_p = \frac{1}{2\pi\sqrt{LC}}\sqrt{1 - \frac{R_l^2 C}{L}}$$

$$f_p = f_s\sqrt{1 - \frac{R_l^2 C}{L}}$$

Practical Parallel Resonance Circuit

1. Maximum impedance

- At f = fp the input impedance of a parallel resonant circuit will be near its maximum value but not quite its maximum value due to the frequency dependence of Rp .

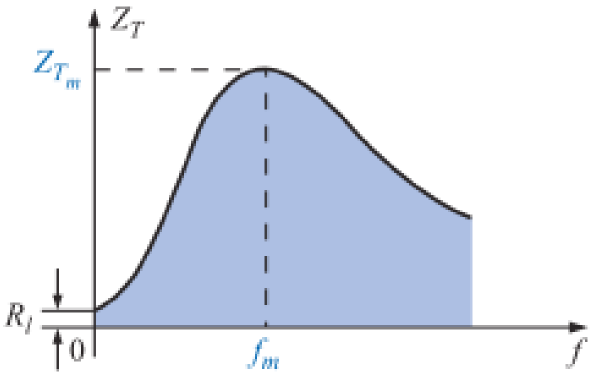


FIG. 20.26
Z_T versus frequency for the parallel resonant circuit.

- The frequency at which maximum impedance will occur is:

$$f_m = f_s\sqrt{1 - \frac{1}{4}\left(\frac{R_l^2 C}{L}\right)}$$

fm is determined by differentiating he general equation for ZT with respect to frequency

2. Minimum impedance

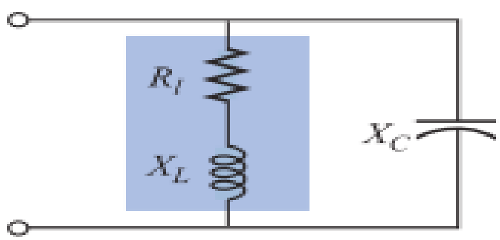


FIG. 20.22
Practical parallel L-C network.

At f = 0 Hz,

Xc is O.C, XL = zero

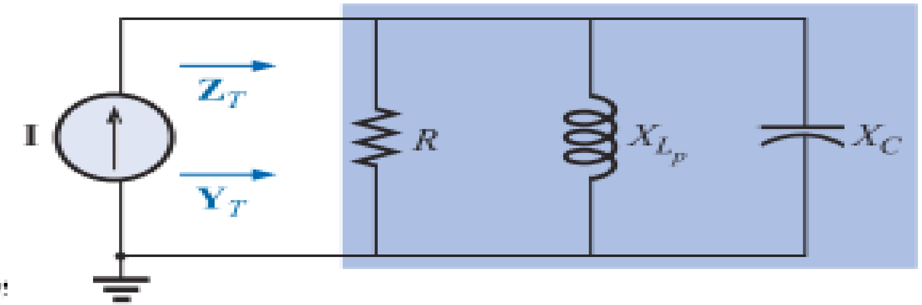
$$Z_T = R_s \parallel R_l \cong R_l.$$

As Rs is sufficiently large for the current source (ideally infinity)

Practical Parallel Resonance Circuit

➢ The quality factor of the practical parallel resonant circuit

determined by the ratio of the reactive power to the real power at resonance



$$Q_p = \frac{V_p^2/X_{L_p}}{V_p^2/R}$$

$$R = R_s \parallel R_p$$

V_p is the voltage across the parallel branches.

$$Q_p = \frac{R}{X_{L_p}} = \frac{R_s \parallel R_p}{X_{L_p}}$$

$$Q_p = \frac{R_s \parallel R_p}{X_C}$$

For the ideal current source ($R_s = \infty\ \Omega$)

$$R = R_s \parallel R_p \cong R_p$$

$$Q_p = \frac{R_s \parallel R_p}{X_{L_p}} = \frac{R_p}{X_{L_p}} = \frac{(R_l^2 + X_L^2)/R_l}{(R_l^2 + X_L^2)/X_L}$$

$$Q_p = \frac{X_L}{R_l} = Q_l$$

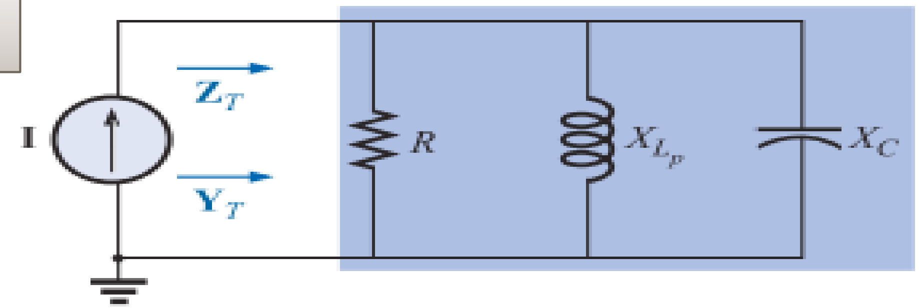
$R_s \gg R_p$

which is simply the quality factor Q_l of the coil.

Practical Parallel Resonance Circuit

- Bandwidth and Half-Power point

$$BW = f_2 - f_1 = \frac{f_r}{Q_p}$$



- The cutoff frequencies f1 and f2 can be determined using the equivalent network shown in the figure:

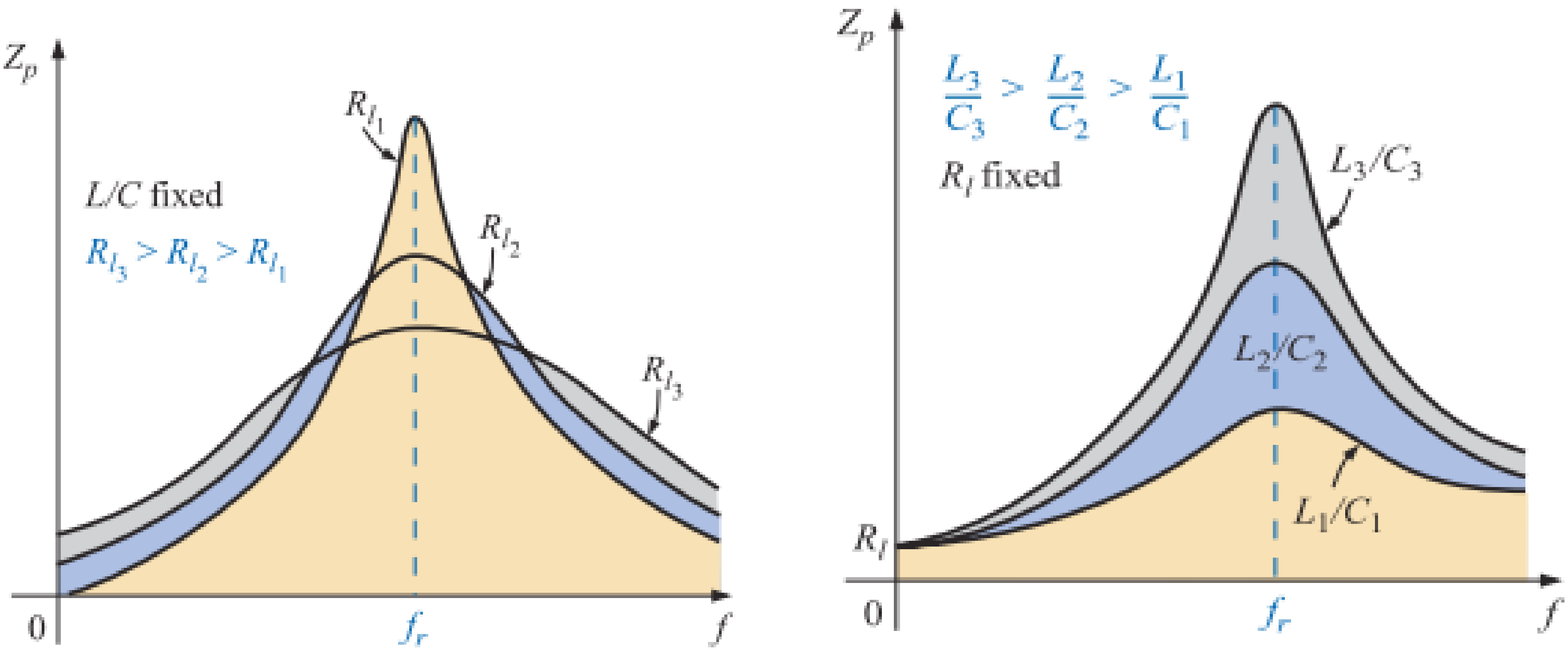
$$\mathbf{Z} = \frac{1}{\frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)} = 0.707R$$

$$f_1 = \frac{1}{4\pi C}\left[\frac{1}{R} - \sqrt{\frac{1}{R^2} + \frac{4C}{L}}\right]$$

$$f_2 = \frac{1}{4\pi C}\left[\frac{1}{R} + \sqrt{\frac{1}{R^2} + \frac{4C}{L}}\right]$$

Practical Parallel Resonance Circuit

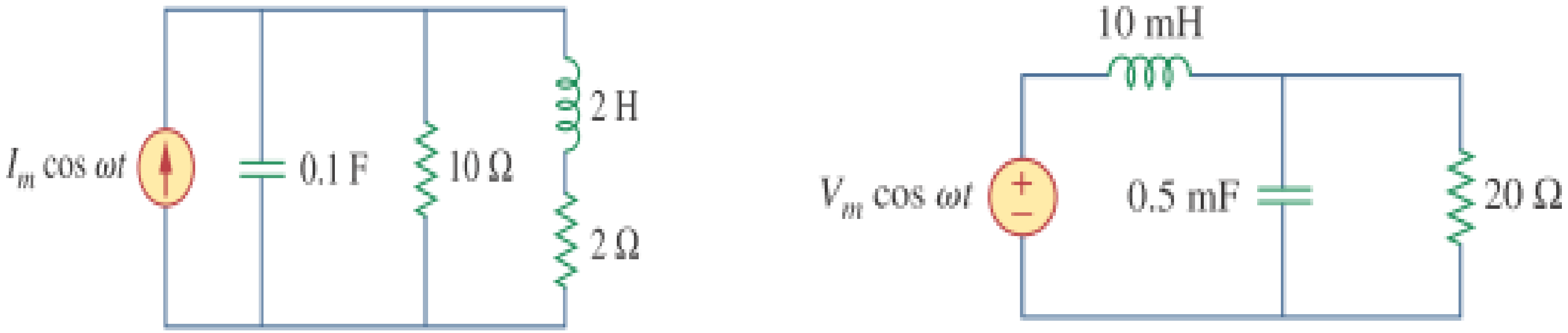
The effect of R_l, L, and C on the shape



- The same effect as in series resonance

Assignment No(1)

Determine the resonant frequency of the circuit in Fig.



- **Validate your analysis using simulation (Proteus or Multisim)**

- **Hint (1): review properties of resonant case to know how to validate the analysis using simulation**
- **Hint (2): you may use "current probe" + Oscilloscope in Multisim**
- **Hint (3): you may use "current probe" + Mixed Graph in Proteus**

1. **Group solution is not permitted**
2. **Cheating or copying other students work will not be tolerated)**

Applications of Resonance Circuits

FILTER NETWORKS

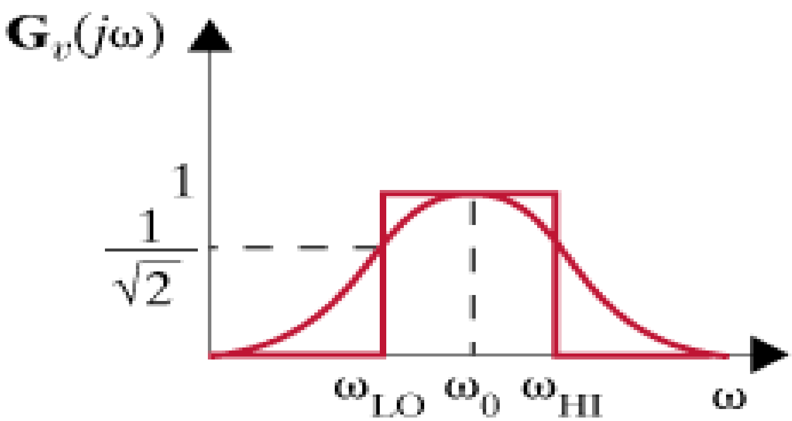
A **filter** is a circuit that is designed to pass signals with desired frequencies and reject or attenuate others.

- **The filter can be treated as a networks designed to have frequency selective behavior**
- **A filter can be used to limit the frequency spectrum of a signal to some specified band of frequencies.**
- **Filters are the circuits used in radio and TV receivers to allow us to select one desired signal out of a multitude of broadcast signals in the environment**

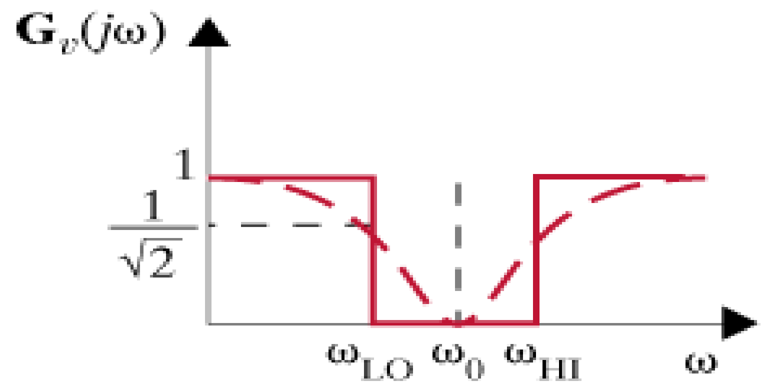
1. **Passive filter:** it consists of only passive elements R, L, and C.
2. **Active filter:** it consists of active elements (such as transistors and op amps) in addition to passive elements

Filter Networks

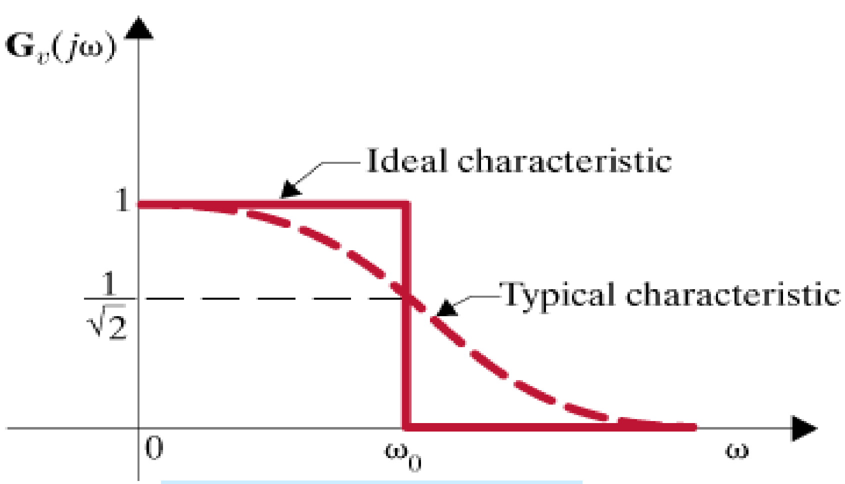
COMMON types of FILTERS



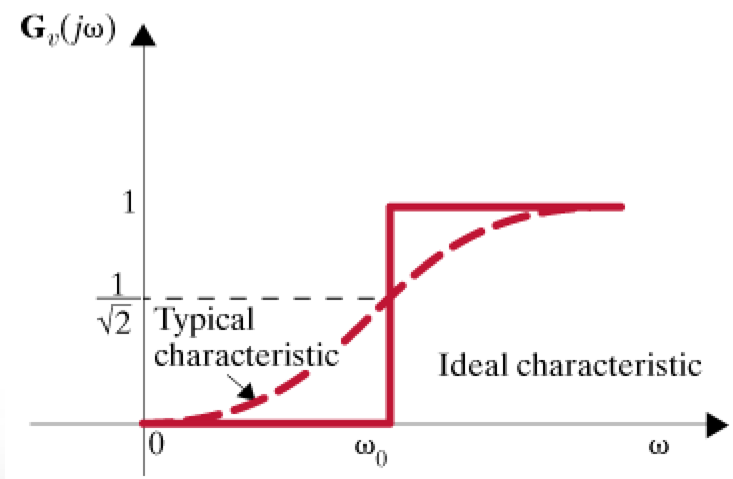
Band-pass filter



Band-reject filter



Low-pass filter



High-pass filter

Filter Networks

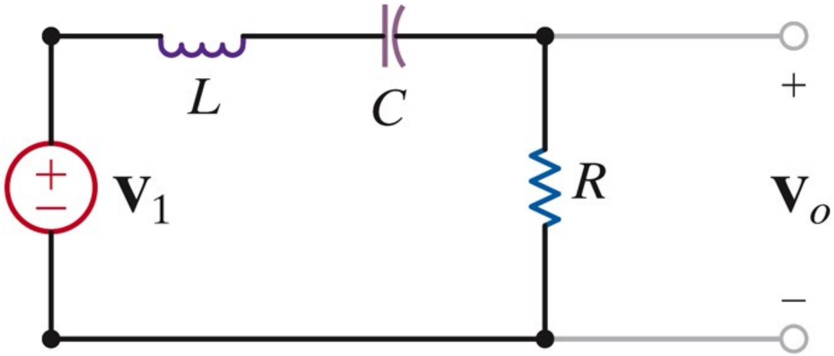
Important Table for determining the type of the filter from its Transfer function

Summary of the characteristics of ideal filters.

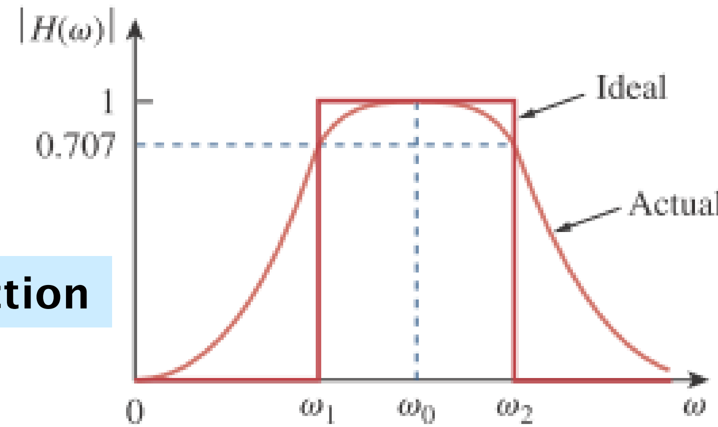
Type of Filter	$H(0)$	$H(\infty)$	$H(\omega_c)$ or $H(\omega_0)$
Lowpass	1	0	$1/\sqrt{2}$
Highpass	0	1	$1/\sqrt{2}$
Bandpass	0	0	1
Bandstop	1	1	0

Filter Networks

Simple Passive band-pass filter



The RLC series resonant circuit provides a bandpass filter when the output is taken off the resistor as shown in Fig.



$$H = \frac{V_0}{V_1} = \frac{R}{R + j\left(\omega L - \dfrac{1}{\omega C}\right)}$$

Transfer Function

$$|H(\omega)| = \frac{\omega RC}{\sqrt{(\omega RC)^2 + (\omega^2 LC - 1)^2}}$$

$$\omega_{LO} = \frac{-(R/L) + \sqrt{(R/L)^2 + 4\omega_0^2}}{2}$$

$$H\left(\omega = \frac{1}{\sqrt{LC}}\right) = 1$$

$$H(\omega = 0) = H(\omega = \infty) = 0$$

$$\omega_{HI} = \frac{(R/L) + \sqrt{(R/L)^2 + 4\omega_0^2}}{2}$$

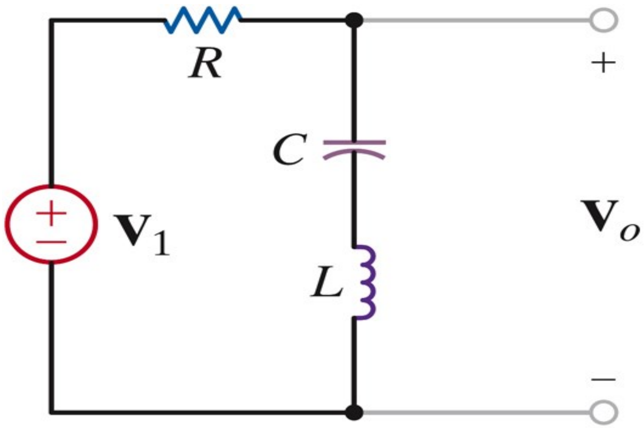
$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$H(\omega_{LO}) = \frac{1}{\sqrt{2}} = M(\omega_{HI})$$

$$BW = \omega_{HI} - \omega_{LO} = \frac{R}{L}$$

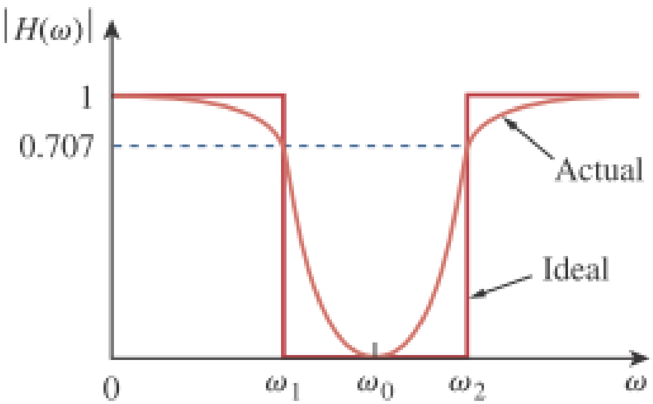
Filter Networks

Simple Passive band-stop filter



- The RLC series resonant circuit provides a band-stop filter when the output is taken off the LC as shown in Fig.
- A filter that prevents a band of frequencies between two designated values
- It is also known as a band-stop, band-reject, or notch filter.

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{j(\omega L - 1/\omega C)}{R + j(\omega L - 1/\omega C)}$$



$$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow j\left(\omega_0 L - \frac{1}{\omega_0 C}\right) = 0$$

at $\omega = 0$ the capacitor acts as open circuit $\Rightarrow V_0 = V_1$

at $\omega = \infty$ the inductor acts as open circuit $\Rightarrow V_0 = V_1$

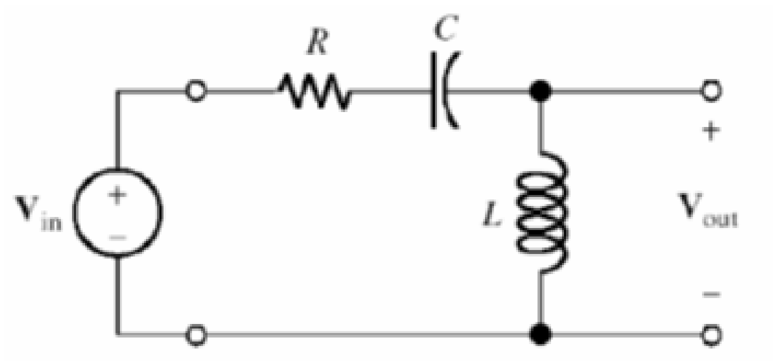
BW = Bandwidth of rejection

$\omega_1,\ \omega_2$ are determined as in the band - pass filter

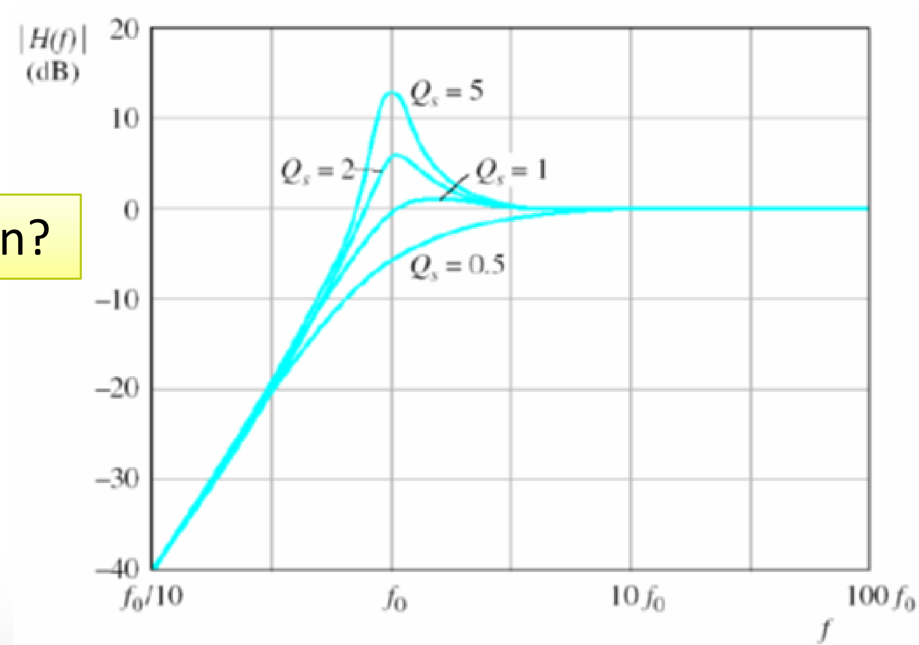
$$\mathbf{H}(0) = 1, \quad \mathbf{H}(\infty) = 1.$$

Filter Networks

High-Pass Filter



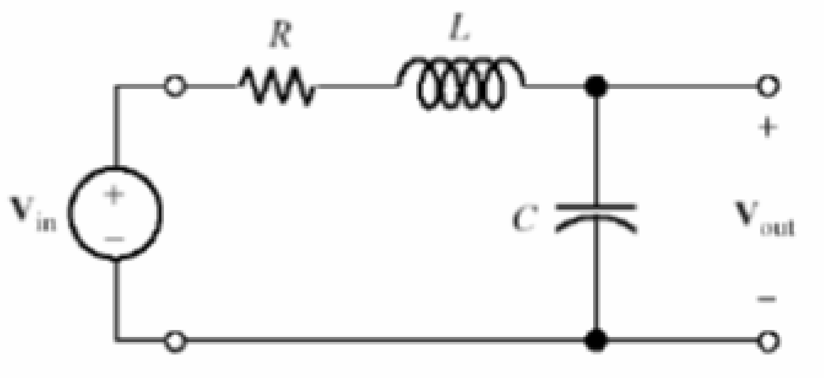
- At low frequency the capacitor is an open circuit (Vo = 0)
- At high frequency the capacitor is a short and the inductor is open (Vo = Vin)



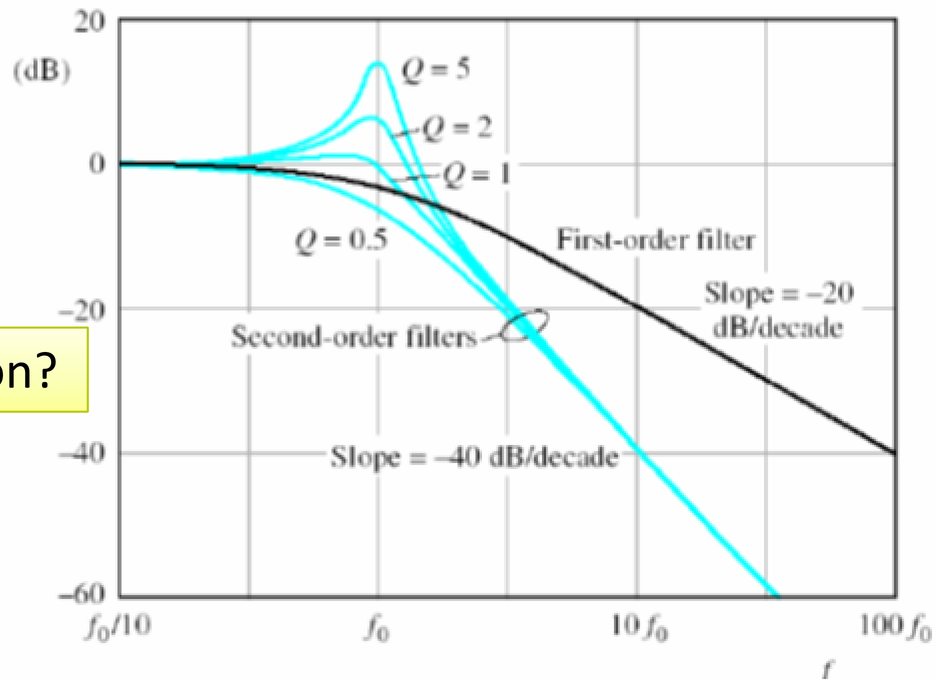
Calculate/ Search for the transfer function?

Filter Networks

Low-Pass Filter



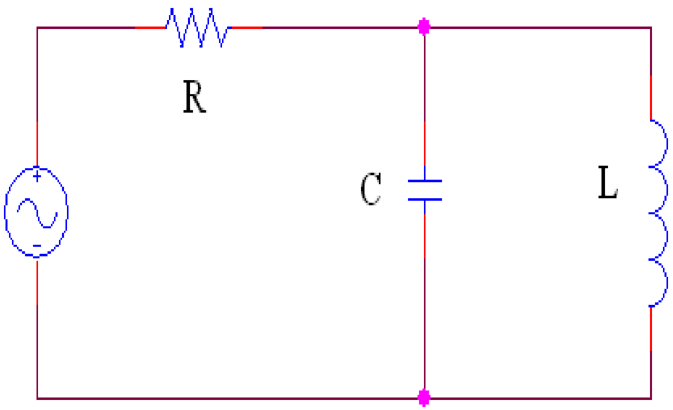
- At low frequency the capacitor is an open circuit (Vo = Vin)
- At high frequency the capacitor is a short and the inductor is open (Vo = 0)



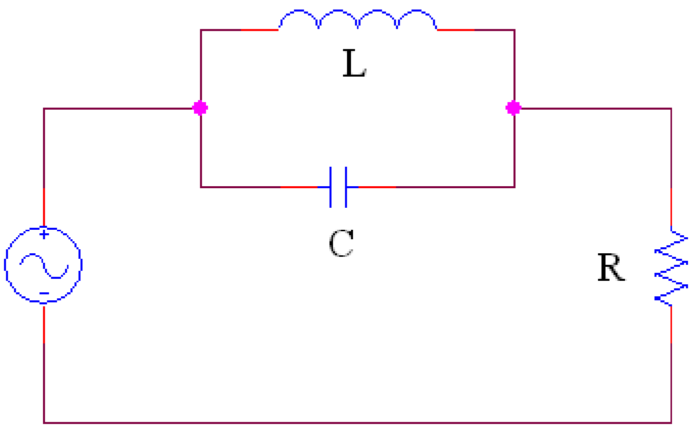
Calculate/ Search for the transfer function?

Filter Networks

Band-Pass Filter (Using Parallel Resonance Circuits)



Band-stop Filter (Using Parallel Resonance Circuits)



Calculate/ Search for the transfer function?

Order of the Filter

- All the previously described filtered are Second Order Filters because they contain two reactive elements (L and C)
- It is possible to create another type of filters using RC or RL only (First-order Filters) >>> not a complete series/parallel resonant circuit

Active Filters

Passive filters have several limitations:

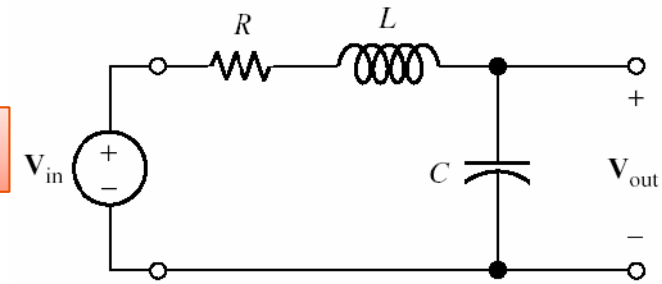
1. **Cannot generate gains greater than one**
2. **Loading effect makes them difficult to interconnect**
3. **Use of inductance makes them difficult to handle**

- **Using operational amplifiers one can design all basic filters, and more, with only resistors and capacitors**

Thank you

Filter Networks

Second Order Low-Pass Filter Transfer Function (1/2)

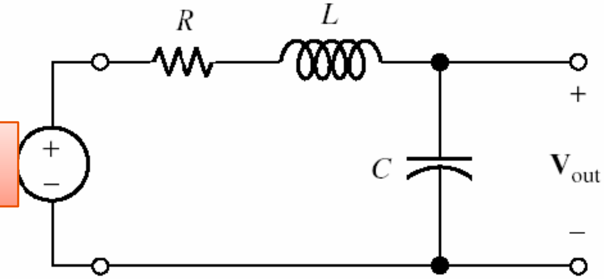


$$\mathbf{V}_{out} = \frac{Z_C}{Z_R + Z_L + Z_C}\mathbf{V}_{in} = \frac{\dfrac{-j}{2\pi fC}}{R + j2\pi fL - \dfrac{j}{2\pi fC}}\mathbf{V}_{in} = \frac{\dfrac{-j}{2\pi fRC}}{1 + j\dfrac{2\pi f_0 L}{R}\left(\dfrac{f}{f_0} - \dfrac{1}{2\pi f f_0 LC}\right)}\mathbf{V}_{in}$$

$$\frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = H(f) = \frac{\dfrac{-j}{2\pi fRC}}{1 + j\dfrac{2\pi f_0 L}{R}\left(\dfrac{f}{f_0} - \dfrac{1}{2\pi f f_0 LC}\right)} = \frac{-jQ_S(f/f_0)}{1 + jQ_S\left(\dfrac{f}{f_0} - \dfrac{f_0}{f}\right)}$$

Filter Networks

Second Order Low-Pass Filter Transfer Function (2/2)



$$H(f) = \frac{\mathbf{V}_{\text{out}}}{\mathbf{V}_{\text{in}}} = \frac{-jQ_s(f_0/f)}{1 + jQ_s(f/f_0 - f_0/f)}$$

$$= \frac{Q_s(f_0/f)\angle -90^\circ}{\sqrt{1+Q_S^2(f/f_0 - f_0/f)^2}\angle Tan^{-1}Q_s(f/f_0 - f_0/f)}$$

$$|H(f)| = \frac{Q_s(f_0/f)}{\sqrt{1+Q_S^2(f/f_0 - f_0/f)^2}}$$